Hierarchical Multi-Objective Planning For Autonomous Vehicles

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- S. Bopardikar, B. Englot and A. Speranzon, “Robust Belief Roadmap: Planning Under Uncertain And Intermittent Sensing”, To be published at ICRA 2014

- X. Ding, A. Pinto and A. Surana, “Strategic Planning under Uncertainties via Constrained Markov Decision Processes”, Appeared in ICRA 2013
Problem: High-Level Mission Specifications

*Autonomous* missions in uncertain environments require:

1) Support optimization over multiple costs
2) Handle logical/spatial/temporal constraints
3) Deal with contingencies at multiple temporal and spatial scales

Mission (example):
Starting from **START**, go to **PICKUP** location, then go one of the **DROPOFF** locations before heading back to **START**. Minimize the expected time of arrival with the constraints that the mission can be accomplished with at least 60% probability and total threat exposure is less than 0.4

*Mission + Motion* Planning
Mission VS Motion Planning

“Starting from **START**, go to **PICKUP** location, then go one of the **DROPPOFF** locations before heading back to **START**. Minimize the expected time of arrival with the constraints that the mission can be accomplished with at least 60% probability and total threat exposure is less than 0.4”

Mission level planning:

- Reach some locations (**START** **PICKUP** **DROPPOFF**)
- Optimize a primary goal (expected time) and satisfy constraints (probability of mission success and threat exposure)

Motion level planning:

- “Figure out” how to do execute the above in a complex city-like environment flying low between buildings to keep coverage
- Ensure that you are generating trajectories that are compatible with the underlying vehicle dynamics
World Model for Mission

Labelled Markov Decision Process

- $q_i$ is the state (facet, orientation)
- $\sigma_i$ is the action
- $P(q_i, \sigma_i, q_j)$ is the probability of transition
- $b_i$ label at state $i$
- $g_i(s_i, \sigma_i)$ is the cost of the action $\sigma_i$
Mission Level Planning

- Given a mission specification expressed as linear temporal logic (LTL) obtain Deterministic Finite State Automaton (DFA)

Starting from **START**, go to **PICKUP** location, then go one of the **DROPOFF** locations before heading back to **START**

\[
\phi = \neg F \text{Fail} \cup (F \text{Start} \land F \text{Pickup} \land F \text{Dropoff} \\
\land G (\text{Start} \rightarrow X F \text{Pickup} \land \text{Dropoff} \rightarrow X F \text{Start}))
\]

- MDP represents the world, the actions and the costs

- Combine the MDP and DFA to obtain a CMDP

- Solve CMDP to obtain (randomized) mission level policy (plan)
Motion Level Planning

- Responsible to execute the mission level policy at a lower level
- Use of evidence grid to represent occupied/unoccupied space
- How do we ensure that there is “consistency” between the mission level planning cost and constraints and the low-level planning objective?
Hierarchical Planning

- Costs and constraints between the different levels of the hierarchy are in correspondence across layers.
Probabilistic Roadmaps

- Samples can be drawn in a deterministic or in a stochastic fashion
- Useful for planning in higher dimensional spaces - e.g. in 3D considering $(x, y, \theta)$ or 6D considering position $(x, y, z) + \text{velocity} (v_x, v_y, v_z)$
- PRM sampling methods are probabilistic complete

1. Randomly sample the configuration space
2. Remove samples that are not collision free
3. Determine path compatible with vehicle dynamics that connects the nodes
4. Connect Start and Goal to closest nodes
Multi-objective Path Planning

- We are interested to compute a plan that minimizes two costs functions $C(\cdot)$ and $Q(\cdot)$.

- To pose this problem we consider the cost function $C(\cdot)$ as primary cost and $Q(\cdot)$ as a secondary cost (constraints) and pose the following problem where now $b$ is considered a free variable.

\[
\min_{\pi \in \mathcal{R}} C(\pi)
\]

\[
\text{ s.t. } Q(\pi) \leq b
\]

- One obtains the full Pareto curve.

- For monotonic non-decreasing costs this graph can be search very efficiently.

Multi-Objective Planning Under Localization Constraints

- We are interested in a multi-objective problem where the secondary cost is a state dependent function.

- In particular, taking into account strong priors, determine a path that minimizes length and position accuracy (never exceeding a maximum).

**Problem:**

\[
\min_{\pi_{sd} \in \mathcal{P}_{sd}} C(\pi_{sd}) \\
\text{s.t.} \quad \lambda\left( P(\pi_{sk}) \right) \leq p_{\text{max}}, \forall \pi_{sk} \subseteq \pi_{sd} \in \mathcal{P}_{sd},
\]

Cov. along path
Planning in Belief Space

This problem is related to work at MIT by Prof. Roy group

- Single objective:
  - Trace of the state estimate error covariance
  - Propagate the EKF over paths
  - Minimize uncertainty at the goal state

- Covariance factorization for fast computation:

  Write $P_t = B_tC_t^{-1}$ as

  $\begin{pmatrix} B_t \\ C_t \end{pmatrix} = \begin{pmatrix} F_t \\ M(\gamma_t)F_t \end{pmatrix} F_t^{-T} \begin{pmatrix} Q_t F_t^{-T} \\ F_t^{-T} + M(\gamma_t)Q_t F_t^{-T} \end{pmatrix} \begin{pmatrix} B_{t-1} \\ C_{t-1} \end{pmatrix}$

- Computation intensive as these weight matrix need be computed across the roadmap
Problem Setup

- We consider a general vehicle and sensing model

\[ x(t + 1) = f(x(t), n(t)) \]
\[ y_j(t) = h_j(x(t), v_j(t)), \quad \forall j \in \{1, \ldots, m(x, t)\}, \]

- The error covariance for the Extended Kalman Filter

\[
P_{t+1}^{-1} = (F_t P_t F_t^T + Q_t)^{-1} + \sum_{j=1}^{m} H_j R_j^{-1} H_j^T,
\]

\[ F, H : \text{Jacobians}, \quad Q, R : \text{Process/Measurement cov.} \]

- We assume:
  - Data association is perfect and no misdetection
  - Consistency (mean state close to planned trajectory)

- To alleviate the computation burden of associate to each edge a matrix and propagate matrices over the edges we consider the \textit{maximum eigenvalue} of the covariance matrix \( \bar{\lambda}(P_t) \)
Maximum Eigenvalue Bound

- Given a set of vertices in the roadmap
- Given a strong prior about the environment

**Theorem**

\[
\overline{\lambda}(P_T) \leq b_\kappa - \zeta + 1 \left( \frac{d - \zeta c}{\zeta c + 1} \right)^{T - \kappa} \frac{1}{\zeta + \lambda(P_0)} + \frac{c}{\zeta c + 1} \left( \frac{1 - (d - \zeta c)^{T - \kappa}}{(\zeta c + 1)^{T - \kappa}} \right)
\]

*where*

\[
b := \overline{\lambda}(Q), \quad c := \inf_{\xi \in X \setminus X_S} \lambda(H(f(\hat{x}, 0))' R^{-1} H(f(\hat{x}, 0)))
\]

\[
d := bc + 1, \quad \zeta := (bc + \sqrt{b^2 c^2 + 4bc})/(2c), \quad \kappa := |X_S|
\]
Multi-Objective Planning with Localization Constraints

- The problem we are interested in is the following:

\[
\min_{\pi_{sd} \in P_{sd}} C(\pi_{sd}) \\
\text{s.t.} \quad \bar{\lambda}(P(\pi_{sk})) \leq p_{\text{max}}, \quad \forall \pi_{sk} \subseteq \pi_{sd} \in P_{sd},
\]

- We can consider a similar approach as discussed before, i.e. solving the problem on an extended graph:
Simulations Results

- Sensor modalities: IMU + LIDAR to range to building corners

The extended graph can become very large

- Planning in a $1km^2$ environment
- 100 vertices on the PRM
- ~2000 edges

<table>
<thead>
<tr>
<th>$p_{\text{max}}$</th>
<th>Uniform Edges</th>
<th>Uniform Nodes</th>
</tr>
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<tbody>
<tr>
<td>8</td>
<td>174542</td>
<td>9257</td>
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<tr>
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<td>15436</td>
</tr>
<tr>
<td>14</td>
<td>359192</td>
<td>16682</td>
</tr>
</tbody>
</table>

How does one choose the quantization level for the secondary cost?
Consider the change of $\bar{\lambda}(P_0)$ over an edge $e \in R$

$$\Delta_e(\bar{\lambda}(P_0)) := B_e(\bar{\lambda}(P_0)) - \bar{\lambda}(P_0)$$

then for each edge $e$ we can compute the worst-case difference $\Delta_e^*$ as this function is concave

$$\delta := \min_{e \in E} \{|\Delta_e^*| : |\Delta_e^*| > 0\}$$
Two Schemes

- **Uniform** \( \delta := \min_{e \in E} \{|\Delta^*_e| : |\Delta^*_e| > 0\} \)

- **Adaptive** \( \delta_j := \min_{i \in N_{\text{fin}}(j)} \{|\Delta^*_{e_{ij}}| : |\Delta^*_{e_{ij}}| > 0\} \)
Results for Adaptive Scheme

- Sensor modalities: IMU + LIDAR to range to building corners

The extended graph can become very large

- Planning in a $1km^2$ environment
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Interactions

- Recall the hierarchical planning

\[
\text{Level I: } \min_{\pi \in \text{Paths}(S,D)} \sum_{t=0}^{\infty} f_1(s_t, u_t)
\]
\[\text{s.t. } \sum_{t=0}^{\infty} g(s_t, u_t) \leq \gamma_2
\]
\[\sum_{t=0}^{\infty} f_2(s_t, u_t) \leq \gamma_3
\]
\[\sum_{t=0}^{\infty} f_3(s_t, u_t) \leq \gamma_4
\]
\[\ldots
\]
\[\sum_{t=0}^{\infty} f_n(s_t, u_t) \leq \gamma_n
\]
\[P^\rho(\text{Mission} \geq p_{\text{success}})
\]

\[
\text{Level II: } \min_{\pi \in \text{Paths}(S,D)} \sum_{t=0}^{T} J(u_t, x_t, \pi)
\]
\[\text{s.t. } f_2(\pi) \leq \Gamma_2
\]
\[f_3(\pi) \leq \Gamma_3
\]
\[f_4(\pi) \leq \Gamma_4
\]
\[\ldots
\]
\[x = f(x, u)
\]
\[x \in X, u \in U
\]
\[\pi \cap \text{Obs} = \emptyset
\]

\[
\text{Level III: } \min_{u_t \in U} \int_0^T f(u_t, x_t, \pi)
\]
\[\text{s.t. } f_3(\pi) \leq \delta_3
\]
\[\ldots
\]
\[x = g(x, u)
\]
\[x \in \bar{X} \supseteq X, u \in \bar{U} \supseteq U
\]
\[g(x, u)(x_{t+1}) = f(x, u)
\]
Example: Interaction Between Mission and Motion Planning

1. Mission planning determine optima policy to have autonomous system go from Start to Goal with constraints on missions success and threat exposure.

2. When new threats are found, interaction between planners lead to a new mission level policy.
Conclusions

- Hierarchical Planning
  - Mission planning from LTL specifications define a policy at coarse scale
  - Motion planning enables navigation in complex environments
  - “Coupled” multi-objective planning algorithms enable autonomous vehicle to deal with contingencies at multiple temporal and spatial scales

- Multi-objective path planning
  - Developed a new algorithms that find a path in a complex environment that minimizes multiple costs
  - Explored computation/accuracy tradeoffs to ensure algorithms can be implemented in real-time.
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