Managing systemic risk

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“Ford went to Capitol Hill in late 2008 pushing for the rescue of its rivals, GM and Chrysler ... GM received $49.5 billion,... Chrysler Group received $10.5 billion in bailout funds”
Networks are important!

“Ford went to Capitol Hill in late 2008 pushing for the rescue of its rivals, GM and Chrysler ... GM received $49.5 billion, ... Chrysler Group received $10.5 billion in bailout funds”

“Without financing during bankruptcy, GM and Chrysler would have had to go out of business, taking down many suppliers. That would have likely caused bankruptcies at the healthier automakers such Ford Motor, who would not have been able to get the parts they needed to build cars.”

– CNN
Systemic Risk

‘system’ ≡ collection of ‘entities’.

Examples:
- firms in an economy
- business units in a company
- suppliers, sub-contractors, etc. in a supply chain network
- generating stations, transmission facilities, etc. in a power network
Systemic Risk

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- generating stations, transmission facilities, etc. in a power network

Systemic risk refers to the risk of the entire system. Involves:

- the simultaneous analysis of outcomes across all entities in a system
- the possibility of complex interactions across the network
None of these firms was weakened by its exposure to Lehman or anyone else. They were weakened by the fact that virtually all of them held—or were suspected of holding—large amounts of what the media came to call toxic assets.
"None of these firms was weakened by its exposure to Lehman or anyone else. They were weakened by the fact that virtually all of them held – or were suspected of holding – large amounts of what the media came to call toxic assets."
Three different approaches

Risk management

- Portfolios known: distributions known but realizations unknown
- Goal: Apportion total risk to various entities

Stylized dynamic models

- Portfolio update rules known
- Goal: Understand the characteristics of resilient networks

Feedback analysis

- Signed directed graphs (SDG) to model feedback
- Goal: Analysis of particular networks
Risk management

$\mathcal{F} \equiv$ set of nodes in the network (firms, suppliers, edges in a graph)

$\tilde{X}_i =$ random loss of node $i$  

$\tilde{X}_\mathcal{F} =$ random loss of the network
Risk management

\( \mathcal{F} \equiv \text{set of nodes in the network (firms, suppliers, edges in a graph)} \)

\( \tilde{X}_i = \text{random loss of node } i \quad \tilde{X}_\mathcal{F} = \text{random loss of the network} \)

**Goal:** Measure for the “acceptability” of \( \tilde{X}_\mathcal{F} \)

- risk measure \( \rho(\cdot) : \rho(\tilde{X}_\mathcal{F}) \) is the “risk” of \( \tilde{X}_\mathcal{F} \)
- Allocate \( \rho(\tilde{X}_\mathcal{F}) \) to individual entities \( i \)
- Incentive compatibility
Examples of Financial Systemic Risk Measures

- $\mathcal{F} =$ firms in the economy
- $X_{i,\omega} =$ loss of a firm $i$ in scenario $\omega$

**Example.** (Systemic Expected Shortfall)

$$\text{CVaR}_\alpha \left( \left\{ \sum_{i \in \mathcal{F}} X_{i,\omega} \right\} \right)$$

[Acharya et al., 2010; Brownlees, Engle 2010]

**Example.** (Deposit Insurance)

$$E^* \left[ \sum_{i \in \mathcal{F}} X_{i,\omega}^+ \right]$$

[e.g., Lehar, 2005; Huang et al., 2009]
## Systemic Risk Measures

| Scenario | Firm 1 | Firm 2 | Firm |\(|\mathcal{F}|\) |
|----------|--------|--------|------|------------------|
| \(\omega_1\) | \(X_{1,\omega_1}\) | \(X_{2,\omega_1}\) | \(X_{|\mathcal{F}|,\omega_1}\) |
| \(\omega_2\) | \(X_{1,\omega_2}\) | \(X_{2,\omega_2}\) | \(X_{|\mathcal{F}|,\omega_2}\) |
| \(\vdots\) | \(\vdots\) | \(\vdots\) | \(\vdots\) |
| \(\omega_{|\Omega|}\) | \(X_{1,\omega_{|\Omega|}}\) | \(X_{2,\omega_{|\Omega|}}\) | \(X_{|\mathcal{F}|,\omega_{|\Omega|}}\) |

0 \(\rightarrow\) \(T\)

\[\Omega = \text{set of scenarios}\]

\[\mathcal{F} = \text{set of firms (entities in the system)}\]

\[X \in \mathbb{R}^{\Omega \times \mathcal{F}}\]

\[X_{i,\omega} = \text{loss for firm} \ i \ \text{in scenario} \ \omega\]
Example

- 3 firms in 3 future scenarios (equally likely)
- Loss matrix (+ Loss; - Profit)

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Questions:
- What is the total “risk” of the economy?
- How does one “allocate” this risk to each of the three firms?
Systemic Risk Measures: Definition

- $\Omega =$ set of scenarios  \hspace{0.5cm} $\mathcal{F} =$ set of firms (entities in the system)
- $X_{i,\omega} =$ loss for firm $i$ in scenario $\omega$ \hspace{0.5cm} $X \in \mathbb{R}^{\Omega \times \mathcal{F}}$
- $X_\omega =$ loss vector in scenario $\omega$
Systemic Risk Measures: Definition

- $\Omega =$ set of scenarios  \hspace{1cm} $\mathcal{F} =$ set of firms (entities in the system)
- $X_{i,\omega} =$ loss for firm $i$ in scenario $\omega$  \hspace{1cm} $X \in \mathbb{R}^{\Omega \times \mathcal{F}}$
- $X_{\omega} =$ loss vector in scenario $\omega$

**Definition.** A **systemic risk measure** $\rho: \mathbb{R}^{\Omega \times \mathcal{F}} \rightarrow \mathbb{R}$ satisfies:

(i) **Monotonicity:** if $X \geq Y$, then

$$\rho(X) \geq \rho(Y)$$

(ii) **Positive homogeneity:** for all $\alpha \geq 0$,

$$\rho(\alpha X) = \alpha \rho(X)$$

(iii) **Normalization:** $\rho(1_\mathcal{F}) = |\mathcal{F}|$
Systemic Risk Measures: Definition

Given $x, y \in \mathbb{R}^F$, define the ordering $x \succeq_\rho y$

$$x \succeq_\rho y \iff \rho \left( \begin{bmatrix} x^T \\ x^T \\ \vdots \\ x^T \end{bmatrix} \right) \succeq \rho \left( \begin{bmatrix} y^T \\ y^T \\ \vdots \\ y^T \end{bmatrix} \right)$$

i.e. an economy with outcome $x$ (resp. $y$) in all states $\Omega$

**Definition.** (con’t.)

(iv) *Preference consistency:* if $X_\omega \succeq_\rho Y_\omega$ for all scenarios $\omega$, then

$$\rho(X) \succeq \rho(Y)$$
Systemic Risk Measures: Definition

Definition. (con’t.)

(v) **Convexity**: for all $0 \leq \alpha \leq 1$, $\bar{\alpha} = 1 - \alpha$

(a) *Outcome convexity*: if

$$Z = \alpha X + \bar{\alpha} Y$$

then, $\rho(Z) \leq \alpha \rho(X) + \bar{\alpha} \rho(Y)$

(b) *Risk convexity*: if for all scenarios $\omega \in \Omega$,

$$\rho(Z_\omega, \ldots, Z_\omega) = \alpha \rho(X_\omega, \ldots, X_\omega) + \bar{\alpha} \rho(Y_\omega, \ldots, Y_\omega),$$

then, $\rho(Z) \leq \alpha \rho(X) + \bar{\alpha} \rho(Y)$

Two different notions of **diversity**

- One allows cross-subsidization
- Other removes randomness
Systemic Risk Measures: Definition

**Definition.** (con’t.)

1. Outcome convexity: Increasing diversity reduces risk

   \[ X_\omega \xrightarrow{\alpha} \oplus \rightarrow Z_\omega \Rightarrow \rho(Z) \leq \alpha \rho(X) + \bar{\alpha} \rho(Y) \]

2. Risk convexity: Removing randomness reduces risk

   \[ \rho(Z_\omega 1_\Omega^\top) \xrightarrow{\alpha} \rho(X_\omega 1_\Omega^\top) \xrightarrow{\bar{\alpha}} \rho(Y_\omega 1_\Omega^\top) \Rightarrow \rho(Z) \leq \alpha \rho(X) + \bar{\alpha} \rho(Y) \]
Structural Decomposition

**Definition.** An aggregation function is a function $\Lambda : \mathbb{R}^\mathcal{F} \rightarrow \mathbb{R}$ that is monotonic, positively homogeneous, convex, and normalized so that $\Lambda(1^\mathcal{F}) = |\mathcal{F}|$.

Aggregation function: aggregates risk across firms in a given scenario.
**Definition.** An aggregation function is a function $\Lambda : \mathbb{R}^F \rightarrow \mathbb{R}$ that is monotonic, positively homogeneous, convex, and normalized so that $\Lambda(\mathbf{1}_F) = |F|$.

Aggregation function: aggregates risk across firms in a given scenario

**Theorem.** A function $\rho : \mathbb{R}^{\Omega \times F} \rightarrow \mathbb{R}$ is a systemic risk measure with $\rho(\mathbb{R}^{\Omega \times |F|}) = \mathbb{R}$ if, and only if, there exists
- an aggregation function $\Lambda$
- coherent single-firm base risk measure $\rho_0$ such that
  \[
  \rho(X) = (\rho_0 \circ \Lambda)(X) \triangleq \rho_0 \left( \Lambda(X_1), \Lambda(X_2), \ldots, \Lambda(X_{|\Omega|}) \right)
  \]
Example: Economic Systemic Risk Measures

- $\mathcal{F} =$ firms in the economy
- $X_{i,\omega} =$ loss of a firm $i$ in scenario $\omega$

Example. (Systemic Expected Shortfall)

$$
\Lambda_{\text{total}}(x) \triangleq \sum_{i \in \mathcal{F}} x_i, \quad \rho_{\text{SES}}(X) \triangleq (\text{CVaR}_\alpha \circ \Lambda_{\text{total}})(X)
$$

[Acharya et al., 2010; Brownlees, Engle 2010]

Example. (Deposit Insurance)

$$
\Lambda_{\text{loss}}(x) \triangleq \sum_{i \in \mathcal{F}} x_i^+, \quad \rho_{\text{DI}}(X) \triangleq \mathbb{E}^* [\Lambda_{\text{loss}}(X_\omega)] = \mathbb{E}^* \left[ \sum_{i \in \mathcal{F}} X_{i,\omega}^+ \right]
$$

[e.g., Lehar, 2005; Huang et al., 2009]
Example: Resource Allocation

- $\mathcal{A} =$ a set of activities
- $\mathcal{F} =$ a set of capacitated resources
- $X_{i,\omega} =$ shortage of resource $i$ in scenario $\omega$

Aggregation function:

$$\Lambda_{RA}(x) \triangleq \min_{u} \sum_{a \in \mathcal{A}} c_a u_a$$

subject to

$$\sum_{a \in \mathcal{A}} b_{ia} u_a \geq x_i, \quad \forall i \in \mathcal{F}$$

where

- $u_a =$ reduction in level of activity $a$ (decision variable)
- $c_a =$ per-unit cost of reductions in activity $a$
- $b_{ia} =$ per-unit consumption of resource $i$ by activity $a$
Example: Eisenberg-Noe Contagion Model

- $\mathcal{F}$ = firms, who have assets and obligations to each other
- $\Pi_{ij}$ = fraction of the debt of firm $i$ owed to firm $j$
- $x$ = losses on the asset portfolio of firms

Aggregation Function: $\gamma > 1$

$$\Lambda_{CM}(x) \triangleq \min_{y \in \mathbb{R}^F_+, b \in \mathbb{R}^F_+} \sum_{i \in \mathcal{F}} y_i + \gamma \sum_{i \in \mathcal{F}} b_i$$

subject to $b_i + y_i \geq x_i + \sum_{j \in \mathcal{F}} \Pi_{ji} y_j$, $\forall i \in \mathcal{F}$.

where loss $x_i$ in firm $i$ is covered by

- reducing payments by $y_i$
- borrowing $b_i$ from the regulator
Risk attribution

Dual representation for risk $\rho(X)$:

$$\rho(X) = \max_{\bar{\pi}, \Xi} \sum_{i \in \mathcal{F}} \sum_{\omega \in \Omega} \Xi_{i,\omega} X_{i,\omega}$$

subject to

$$(1, \bar{\pi}) \in A^*$$

$$(\bar{\pi}_\omega, \Xi_\omega) \in B^*, \forall \omega \in \Omega$$

$\bar{\pi} \in \mathbb{R}^\Omega$, $\Xi \in \mathbb{R}^{\mathcal{F} \times \Omega}$

Risk attributed of firm $i$: $y_i^* = \sum_{\omega \in \Omega} \Xi_{i,\omega}^* X_{i,\omega}$

**Theorem.** (No Undercut) Given $\alpha \in \mathbb{R}^{|\mathcal{F}|}_+$, define

$$r(\alpha) \triangleq \rho(\alpha_1 x_1; \ldots; \alpha_{|\mathcal{F}|} x_{|\mathcal{F}|})$$

Then,

$$\alpha^\top y^* \leq r(\alpha)$$

Example: Risk Attribution

- 3 firms in 3 future scenarios (equally likely)
- $\rho_{\text{SES}}(X) \triangleq (\text{CVaR}_{1/3} \circ \Lambda_{\text{total}})(X) = \text{CVaR}_{1/3} (x_1 + x_2 + x_3) = 30$

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Structural decomposition extends broadly

**Homogeneous Systemic Risk Measures:**
- monotone, \(+\)vely homogeneous, preference consistent, \textbf{not} convex
- structural decomposition exists
  - Homogeneous single-firm base risk measure
  - Homogeneous aggregation function

**Convex Systemic Risk Measures:**
- monotone, convex, preference consistent, \textbf{not} \(+\)vely homogeneous
- structural decomposition exists
  - convex single-firm base risk measure
  - convex aggregation function

**Key idea:** Preference consistency allows for the structural decomposition
Further extensions

- Coherent systemic risk measures
- Convex systemic risk measures
- Monotone positively homogeneous systemic risk measures

  - $\rho(X) = \inf\{\pi(Y) : \Lambda(X - Y) \in A\}$, $Y = \text{capital injection}$

  - $\rho(X) = \{Y : \Lambda(X + Y) \in A^Y\} \subseteq \mathbb{R}^{\mathcal{F}}$
Asset-firm networks

$X_{\mathcal{F}}$ random losses of $\mathcal{F}$ nodes in $\Omega$ scenarios

- $X_{\mathcal{F}}$ is completely exogenous
- The nodes do not take any actions

The network consists of just the firms. Only captures cross-firm lending

- defaults, hair-cut, funding liquidity

Firms also interact via commonly held assets

- MBS, fire sales, volatility, risk-aversion
Model: Assets, firms and portfolio rules

\( \mathcal{A} = \) set of assets. \( \mathcal{F} = \) set of firms.

- Portfolio rules for firms: \( \Pi \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{F}|} \)

\[ \Pi_{i,h} = \text{fraction of wealth of firm } h \text{ invested in asset } i \]

- Portfolio rules \( \Pi(q, x) \) rules depend on ...
  - prices \( q \)
  - exogenous (risk) factors \( x \)
  - could also depend on wealth \( w \) (in the paper!)
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  - prices $q$
  - exogenous (risk) factors $x$
  - could also depend on wealth $w$ (in the paper!)

Example: CRRA utility with risk aversion $\beta_h$, log-normal payoffs

$\log(p_h) \sim N(\mu_h, \Sigma_h)$.

$$\Pi_h(q, \mu_h, \Sigma_h, r_f) = \frac{1}{\beta_h} \Sigma_h^{-1} \left( \mu_h - \log(q) - r_f 1 + \frac{1}{2} \text{diag}\{\Sigma_h\} \right)$$
Prices are endogenous!

Market clearing implies

\[ q = D^{-1} \Pi(q, x)w = D^{-1} \Pi(q, x)(\theta^0(x) + \Theta^T q) \]
Prices are endogenous!

Market clearing implies

\[ q = D^{-1}\Pi(q, x)w = D^{-1}\Pi(q, x)(\theta^0(x) + \Theta^Tq) \]

Implicit function theorem

\[ \frac{\partial q}{\partial x} = D^{-1}\begin{bmatrix} I - \Pi\Theta^T - HD^{-1} \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial\Pi}{\partial x}w + \Pi\frac{\partial\theta^0}{\partial x} \end{bmatrix} \]

Network Effect \hspace{2cm} Direct Effect

\[ \Delta q = D^{-1}\begin{bmatrix} I - \Pi\Theta^T - HD^{-1} \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial\Pi}{\partial x}w + \Pi\frac{\partial\theta^0}{\partial x} \end{bmatrix} \Delta x \]

Direct Effect, propagated via Network Effect, forms price change.
Network Effect

\[
\frac{\partial q}{\partial x} = D^{-1} \left[ I - \Pi \bar{\Theta}^T - HD^{-1} \right]^{-1} \left[ \frac{\partial \Pi}{\partial x} w + \Pi \frac{\partial \theta^0}{\partial x} \right]
\]

Two components

- \( \Pi \bar{\Theta}^T \in \mathbb{R}^{\left| \mathcal{A} \right| \times \left| \mathcal{A} \right|} \): holding-induced cross-asset interaction
  
  \[
  (\Pi \bar{\Theta}^T)_{ij} = \sum_{h=1}^{\left| \mathcal{F} \right|} \Pi_{ih} \bar{\Theta}_{jh}.
  \]

- \( H \in \mathbb{R}^{\left| \mathcal{A} \right| \times \left| \mathcal{A} \right|} \): wealth-weighted cross-asset portfolio sensitivity
  
  \[
  H_{ij} = \sum_{h=1}^{\left| \mathcal{F} \right|} \frac{\partial \Pi_{ih}}{\partial q_j} w_h.
  \]

Portfolio tracking: \( \Pi = \text{constant} \ldots \ H \equiv 0. \)
Network Effect

\[
[I - \Pi \bar{\Theta}^T]^{-1} = I + \Pi \bar{\Theta}^T + (\Pi \bar{\Theta}^T)^2 + (\Pi \bar{\Theta}^T)^3 + \ldots.
\]

Direct Effect \( = I \cdot [\text{DE}] \)

Primary Network Effect \( = \Pi \bar{\Theta}^T \cdot [\text{DE}] \)

Secondary Network Effect \( = (\Pi \bar{\Theta}^T)^2 \cdot [\text{DE}] \)

\((\Pi \bar{\Theta}^T)^t_{ij} = t\)-th order impact from asset \(j\) to \(i\) over paths of length \(2t\).

\[
\Pi_{ih} \bar{\Theta}_{kh} \cdot \Pi_{kg} \bar{\Theta}_{jg}
\]
Network Design

Decompose $\Pi$ into leverage $b \in \mathbb{R}^{|\mathcal{F}|}$ and holding network $X \in \mathbb{R}^{|\mathcal{A}| \times |\mathcal{F}|}$

- $b_h = \text{leverage for firm } h$
- $X_{ih} = \text{fraction of investment into asset } i$
- $\Pi_{ih} = b_h X_{ih}$

Feasible economies: $(D, q, w, b)$ such that $\mathbf{1}_\mathcal{A}^\top Dq = w^\top b$
Network Design

Set of feasible holding networks

\[ \mathcal{X} = \left\{ X : Dq = X \text{diag}(b)w, \mathbf{1}_A^\top X = \mathbf{1}_F^\top, X \geq 0 \right\} \]

Suppose \( \Theta \) is in equilibrium. Then

\[ \Pi \tilde{\Theta}^\top = Y(X) = X \text{diag}(b) \text{diag}(w) \text{diag}(b) X^\top [D \text{diag}(q)]^{-1}. \]
Network Design

Set of feasible holding networks

\[ \mathcal{X} = \left\{ X : Dq = X \text{diag}(b)w, 1_A^T X = 1_F^T, X \geq 0 \right\} \]

Suppose \( \Theta \) is in equilibrium. Then

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The Maximal Network Amplifier

\[ \text{MNA} \triangleq \rho([NE(X)]) = \rho([I - Y(X)]^{-1}) \]

where \( \rho(\cdot) \) is the spectral radius of a matrix.
Network Design

Set of feasible holding networks

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The Maximal Network Amplifier

\[ \text{MNA} \triangleq \rho([\mathcal{N}E(X)]) = \rho([I - Y(X)]^{-1}) \]

where \( \rho(\cdot) \) is the spectral radius of a matrix.

Network design problem: \( \min_{X \in \mathcal{X}} \text{MNA}(X) \)
Low and High Leverage Regimes

Define: \( \lambda_{\text{max}} \triangleq \min_{X \in \mathcal{X}} \lambda_{\text{max}}(Y(X)) \) \hspace{2cm} \( \lambda_{\text{min}} \triangleq \max_{X \in \mathcal{X}} \lambda_{\text{min}}(Y(X)) \)

Low leverage economy \( \triangleq \lambda_{\text{max}} < 1 \) \hspace{2cm} High leverage economy \( \triangleq \lambda_{\text{min}} > 1 \)
Low and High Leverage Regimes

Define: \[ \lambda_{\text{max}} \triangleq \min_{X \in \mathcal{X}} \lambda_{\text{max}}(Y(X)) \quad \lambda_{\text{min}} \triangleq \max_{X \in \mathcal{X}} \lambda_{\text{min}}(Y(X)) \]

Low leverage economy \( \triangleq \lambda_{\text{max}} < 1 \) 
High leverage economy \( \triangleq \lambda_{\text{min}} > 1 \)

**Theorem:** For any economy

\[ \lambda_{\text{min}} \leq \lambda_{\text{max}} \]

Low leverage and high leverage economies are disjoint.
Theorem: For a low-leverage economy:

\[ \min_{X \in \mathcal{X}} \text{MNA}(X) \leq \frac{1}{1 - \lambda_{\text{max}}} \]

Bound achieved by the mutual-fund network

\[ X^* \triangleq \frac{1}{1^\top A D q} (D q) 1^\top_f \]

In a mutual fund network

- All firms invest in the same portfolio
- The risks of the firms are completely pooled
- Risk management achieved by diversification
Desirable network: **High Leverage Economy**

**Theorem** For a high-leverage economy

\[
\min_{X \in \mathcal{X}} \text{MNA}(X) \leq \frac{1}{\lambda_{\text{min}} - 1}
\]

Bound asymptotically achieved by an **isolated network**

\[
X^* = \begin{pmatrix}
1 & \ldots & 1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\
0 & \ldots & 0 & 1 & \ldots & 1 & 0 & \ldots & 0 \\
0 & \ldots & 0 & 0 & \ldots & 0 & 0 & \ldots & \ldots \\
0 & \ldots & 0 & 0 & \ldots & 0 & 1 & \ldots & 1
\end{pmatrix}
\]

- Firms invest in only one asset
- The firms are clustered into groups that do not interact
- Risk management achieved by **diversity**
Systemic risk management \equiv managing feedback

Systemic risk is a consequence of positive feedback loops.

Networks or directed graphs do not enough information to identify them.
Systemic risk management ≡ managing feedback

Systemic risk is a consequence of positive feedback loops

Networks or directed graphs do not enough information to identify them

Propose signed digraphs (SGD) as the next level of detail

- Used in the process engineering literature
- Extends the analysis from arcs to loops – non-local interactions
- Systematic analysis of the hazards and instabilities
- Compromise between full control theoretic analysis and graphs
Systemic risk management ≡ managing feedback

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Financial entity ≡ processing plant that transforms inputs to outputs
- Graphs are good for flows, e.g. internet, power grid, etc.
- Signed digraphs are good for flow transformations
SDG example: Continuous stirred reactor

Inputs: Concentration $c_{A_i}$ of $A$ and temperature $T_i$
Output: Exothermal Reaction $A \rightarrow B$
Control: Temperature set point $T_{sp}$
SDG example: Continuous stirred reactor

- Solid arc: positive gradient, e.g. $\frac{\partial c_A}{\partial c_{A_i}} > 0$.
- Negative arc: negative gradient, e.g. $\frac{\partial c_A}{\partial r} < 0$.

Loops
- $c_A \rightarrow r \rightarrow c_A$: negative feedback
- $T \rightarrow r \rightarrow T$: positive feedback
- $T \rightarrow \epsilon \rightarrow F_c \rightarrow T$: negative feedback
Simplified bank-dealer network

- Bank/Dealer
  - Prime Broker
    - Loans (cash)
    - Securities as collateral
  - Finance Desk
    - Securities as collateral
    - Cash
  - Trading Desk
    - Trading Desk inventory adjustments (buying and selling securities)
  - Hedge Fund
    - Hedge Fund portfolio adjustments (buying and selling securities)

- Cash Provider (e.g. Money Market Fund)
- Banker/Dealer (OTC) Market
SDG for bank-dealer
SDG for bank-dealer: fire sales
Positive feedback loop

- $P_{BDM} \rightarrow C_{PB} \rightarrow V_{PB} \rightarrow L_{HF} \rightarrow Q_{HF} \rightarrow P_{BDM}$
SDG for bank-dealer: funding runs
SDG for bank-dealer: funding runs

Positive feedback loop

\[ P_{BDM} \rightarrow C_{MM} \rightarrow F_{MM} \rightarrow V_{FD} \rightarrow \lambda^{SP}_{TD} \rightarrow \varepsilon_{TD} \rightarrow P_{BDM} \]
Summary

An axiomatic framework for systemic risk

- Subsumes many recently proposed risk measures
- Structural decomposition of systemic risk
- Methodology extends to a much broader class of risk functions

Structural model for asset-firm contagion

- Endogenous asset prices
- Direct Effect, propagated via Network Effect, forms price change
- low-leverage economies favor mutual fund holding networks
- high-leverage economies favor isolated holding networks

Signed di-graph (SGD) to identify positive feedback loops

- Fast depth first algorithms for discovering loops
- Identifies fire sales and funding runs